

# Comment on “BCS superconductivity of Dirac fermions in graphene layers”

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In reference[1], Kopnin and Sonin (KS) apply the standard BCS model for a two dimensional electron gas with the spectrum of Dirac fermions, namely  $\xi_{\mathbf{p}}^{\alpha} = \alpha p - \mu$ , ( $\alpha = \pm$ ), where  $\mathbf{p}$  is the momentum around the Dirac point and  $\mu$  is the chemical potential. Their attempt is a generic derivation of superconducting properties disregarding microscopic details and the sublattice structure in graphene. In this comment we argue that apart from their derivation of the charge current, the thermodynamic results of ref.[1] are not new and were derived before in ref.[2, 3]; second, we show that the spectroscopic results in ref.[1] such as the superfluid velocity are inconsistent with a Hamiltonian of Dirac fermions. Finally, we show that in spite of the fact that their final result for the current coincides with the correct result for graphene at  $\mu = 0$ , the derivation for Dirac fermions requires regularization, which is only provided by the inclusion of a periodic spectrum in the Hamiltonian.

Ref.[1] starts from the usual BCS spectrum for *s*-wave pairing,  $E_{\mathbf{p}}^{\alpha} = \sqrt{(\xi_{\mathbf{p}}^{\alpha})^2 + |\Delta|^2}$  ( $\alpha = \pm$ ), which is the same spectrum derived in Ref.[2, 3] from a particular model of Dirac fermion superconductivity. The fact that KS do not specify a Hamiltonian, however, does not make their thermodynamic results more general than those previous derivations (as claimed by them) for a trivial reason: since the free energy at the mean field level is defined only by the spectrum[4], any class of BCS fermionic Hamiltonians which share the same spectrum will have *exactly* the same thermodynamic properties. Since KS start from the same BCS spectrum as ref[2, 3], they *should necessarily* obtain the same results for the gap equation and the critical temperature, disregarding any details of the matrix structure in the Hamiltonian. KS describe the results in Eq. (3)–(11) and the subsequent equation as if they corresponded to a new derivation, which is not the case[5].

In the second part of ref.[1], KS calculate the supercurrent,  $\mathbf{j}$ , induced by a uniform flow of the condensate with constant momentum  $\mathbf{k}_s = \nabla\chi$ , where  $\chi = \nabla\chi \cdot \mathbf{r}$  is the phase of the superconductor order parameter,  $\Delta = |\Delta|e^{i\chi}$ . At the charge neutrality point ( $\mu = 0$ ) the Bogoliubov-DeGennes (BdG) equations for a *Dirac* Hamiltonian with *s*-wave pairing are[6]

$$(\mathbf{p} + \mathbf{k}_s) \cdot \vec{\sigma} \hat{u} + \Delta \hat{v} = E \hat{u}, \quad -(\mathbf{p} - \mathbf{k}_s) \cdot \vec{\sigma} \hat{v} + \Delta^* \hat{u} = E \hat{v}, \quad (1)$$

instead of Eq.(2) in ref.[1], where  $\vec{\sigma}$  are *x, y* Pauli matrices. These equations result in a different set

of eigenvectors and also in a different spectrum,  $\sqrt{E_{\mathbf{p}}^2 + k_s^2 \pm 2\sqrt{(\mathbf{p} \cdot \mathbf{k}_s)^2 + k_s^2}|\Delta|^2}$ , with distinct spectroscopic properties for finite  $\mathbf{k}_s$ . We note that due to particle-hole symmetry, the group velocity of the quasi-particles around the Dirac point is zero, whereas the particle-hole charge current is finite[7]. This symmetry argument shows that the spectrum derived in ref.[1] (which gives a finite superfluid velocity at half filling) is inconsistent with *any* BdG Hamiltonian of Dirac fermions, and therefore is not applicable to graphene.

Finally, using a covariant momentum in the BdG Hamiltonian,  $\hat{H}$ , namely  $\mathbf{k}_s = \nabla\chi - \mathbf{A}$ , (or  $\mathbf{k}_s = -\mathbf{A}$  by a gauge choice), where  $\mathbf{A}$  is the vector potential, the current follows from  $\mathbf{j} = -\partial\langle\hat{H}\rangle/\partial\mathbf{A}$ . From Eq. (1) one finds  $\mathbf{j} \propto \{D - 2|\Delta|\tanh[|\Delta|/(2T)]\}\mathbf{A}$ , where  $D \gg |\Delta|$  is the band width, which accounts for the orbital paramagnetic response of the lower band electrons, overwhelming the diamagnetism. This term is absent from the current definition of[1] without justification[8]. The diamagnetism is recovered only if one includes the full spectrum,  $\xi_{\mathbf{p}}^{\alpha} = \alpha|\phi_{\mathbf{p}}| = \alpha|\phi'_{\mathbf{p}} + i\phi''_{\mathbf{p}}|$ , where  $\phi_{\mathbf{p}}$  is a periodic function. In that case, the graphene BdG equations,  $(\phi'_{\mathbf{p}-\mathbf{A}}\sigma_x + \phi''_{\mathbf{p}-\mathbf{A}}\sigma_y)\hat{u} + \Delta \hat{v} = E \hat{u}$ , and  $-(\phi'_{\mathbf{p}+\mathbf{A}}\sigma_x + \phi''_{\mathbf{p}+\mathbf{A}}\sigma_y)\hat{v} + \Delta^* \hat{u} = E \hat{v}$ , will give  $j_i \propto \{S_i + |\Delta|^2 \sum_{\mathbf{p}} |\partial_{p_i} \phi_{\mathbf{p}}|^2 E_{\mathbf{p}}^{-1} \partial_{E_{\mathbf{p}}} [\tanh(E_{\mathbf{p}}/2T)/E_{\mathbf{p}}]\} A_i$ , ( $i = x, y$  directions) where  $S_i$  is a surface term which is regularized by the Brillouin zone[9].

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- [2] A. H. Castro Neto, Phys. Rev. Lett. **86**, 4382 (2001).
- [3] B. Uchoa *et. al.*, Phys. Rev. B **71**, 184509 (2005).
- [4] The free energy for Dirac fermions,  $\Omega$ , is shown in Eq. (24) of ref.[3]. The gap equation follows from  $\partial\Omega/\partial|\Delta| = 0$ .
- [5] For the gap, Eq. (4), (5), (6) in ref.[1] correspond to Eq. (17), (15) and (20) in ref.[3]. For the critical temperature, Eq. (7), (11) and the subsequent equation in[1] correspond to Eq. (A.1), and (23) in ref.[3]. Fig.2 in ref.[1] is equivalent to Fig 5 in ref.[3]. Eq. (5) and (9) in ref.[1] were also derived in ref.[2].
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- [7] X. Yang, and C. Nayak, Phys. Rev. B **65**, 064523 (2002).
- [8] This term can be easily calculated in the normal state,  $\mathbf{j} = \sum_{\mathbf{p}} \langle \Psi_{\mathbf{p}}^{\dagger} \vec{\sigma} \Psi_{\mathbf{p}} \rangle = 2 \sum_{\mathbf{p}, \alpha} \alpha \frac{\mathbf{p} - \mathbf{A}}{|\mathbf{p} - \mathbf{A}|} n(\xi_{\mathbf{p}-\mathbf{A}}^{\alpha}) \propto D\mathbf{A}$ , where  $n$  is the Fermi distribution. The current definition in ref.[1] ignores the lower band of the Dirac cone, and therefore it trivially reproduces the free electron gas (*i.e.* normal

metal) case, where  $\mathbf{j} = 0$ . In contrast with metals, the free energy for Dirac fermions depends on  $D$  [3], and the Ginzburg-Landau current derived from it requires regularization.

[9]  $S_i = -\sum_{\mathbf{P},\alpha} \partial_{p_i} \{\alpha(\partial_{p_i} E_{\mathbf{P}})n(\alpha E_{\mathbf{P}})\}$  is exactly zero when calculated in the whole Brillouin zone, and is  $\propto D$  for a linear (non-periodic) dispersion.